Research Paper Business Analytics

Optimizing the planning of the one day treatment facility of the VUmc

Author: Babiche de Jong

Supervisors: Marjolein Jungman René Bekker

Vrije Universiteit Amsterdam
Faculty of Sciences
De Boelelaan 1081
1081 HV Amsterdam

Host organization:
VU University Medical Center Amsterdam
De Boelelaan 1117
1081 HV Amsterdam

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Preface
As part of the Master’s programme Business Analytics, students are required to produce a research paper. The aim of the research is to perform a small study in the field of Business Analytics. This research should subsequently be described in a clear manner, e.g. concise and ‘to the point’, for the benefit of an expert manager, who has some general expertise in the area of the subject and only limited time.

This research paper, describes how I have constructed an optimization model for the planning of a one day treatment facility of the VUmc. This model is created to help with calculations for what-if scenario’s. It is important to mention that this optimization model is a part of a more complete investigation into the one day treatment facility. Thus, VU University Medical Center will conduct more elaborate research with this optimization model at hand.

I would like to express my sincere gratitude to my supervisor René Bekker for his patience, motivation, and knowledge on the matter. His continuous support, constructive suggestions, and guidance made this paper possible. I would also like to thank Marjolein Jungman who gave me the challenging opportunity of writing this research paper. Her insightful comments and tough questions encouraged me to work even harder. Lastly, this research was supported by the VU University Medical Center Amsterdam. I am very grateful to Corry Mietus, José Stolker and the nursing staff who provided insights and expertise that greatly assisted the research.

Babiche de Jong
Amsterdam, January 2016
Management summary
VU University Medical Center made an organizational decision in July 2016, the merger of two treatment units; the unit for oncology (DBU OHL) and the unit for internal medicine (DBU ALG). The motivation was to have a better distribution and coordination of capacity, as well as being able to anticipate on developments within different specializations. With this merger, VUmc hoped to create a more balanced workload for nurses and enable growth of production. With this new (merged) situation, VU University Medical Center is interested in the optimal capacity and how to optimize the planning of the new (combined) unit, with an optimal balance between the utilization of existing beds and staff, and robustness against disturbance (e.g. emergency patients arriving, lab results, delay earlier in the chain/throughout the day). Thus, the purpose of this research paper is to help answer this by a) developing a mathematical model that can help with calculations for what-if scenarios and b) creating an optimization model.

To get an insight in the current situation, based on the characteristics of each DBU, the provided data is briefly analysed. This resulted in relevant parameters for the model, such as the arrival rate and the length of stay of a patient. Furthermore, the data analysis generated five patient groups based on the actual average duration of the appointment and the bed the treatment was performed on. After the data analysis, a performance analysis was conducted. This gave insight in the way of planning in the current situation. Then a model was implemented based on LP-problems; one for each unit and one for the merged situation. These LP-problems make use of the specification of all patients in the five patient groups. Moreover, to be able to determine the optimal capacity in specific situations the LP-problems make use of infinite server queues (the Mt/G/∞ model). The LP-problems helped with optimizing the planning. Finally, calculations for what-if scenarios were performed, which resulted in an optimal planning for different scenarios with the help of the LP-problems.

The performance analysis showed that the current way of planning the treatments at the outpatient units each day is not as bad as assumed by VUmc. As the average demand for care at DBU OHL lies around 20 whereas 28 beds are available during the day, based on the provided data, it can be concluded that the bed capacity should not be a bottleneck. For DBU ALG the average demand lies around 4 beds throughout the day, whereas 5 beds are available. However, in this study, only the average is considered and not the variance in bed occupancy. From the optimization model, it was seen that it would be most efficient to plan arrivals of patients with a relatively longer appointment duration at the beginning of the day. Furthermore, more arrivals at the beginning of the day means filling up the available beds at the beginning of the day as fast as possible, which might be desirable. Moreover, planning early arrivals would lead to a stable workload for nurses throughout the day, and would make sure that all restrictions set by VUmc are satisfied.

To conclude, creating patient groups based on multiple characteristics, such as type of treatment, length of treatment, whether an appointment requires a consultation meeting prior to the treatment and travelling distance to hospital, might create a more viable model for VUmc. However, one should keep in mind that these groups do not get too refined. Some further research in the direction of the combination between consultation meeting and treatment at the DBU might be interesting as well.
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1 Introduction

1.1 Background information
Over more than 20 years, the number of one day treatments (outpatient treatments) within a hospital has increased immensely. This increase is mainly a consequence of new treatment and research capabilities [1]. Furthermore, the increase is partly caused as more and more treatments no longer require hospitalization, and one day treatment is sufficient. Moreover, from 2001 onwards, more money has been made available to hospitals for the elimination of waiting lists. The latter has produced an increase in both one day treatments and clinical treatments [2].

1.1.1 One day treatment vs. hospitalization
For the purpose of this research, the difference between day (or outpatient) treatment and hospitalization (or inpatient treatment).

- One day treatment (outpatient treatment)
Two general definitions of day treatments are as follows,
“Day treatment is a treatment in an institution, where admission and discharge of the patient are on the same day.” [3]

“The facilities of an institute are only used during the day, and hospitalization of the patient is not required.” [3]

As of 1 January 2016, the definition for one day treatment (or outpatient treatment) reads as follows:
“A form of nursing and/or treatment with a minimum duration of three-hours, on an outpatient equipped department. In general, in connection with the fact that on the same day a medical specialist’s examination or treatment will take place.” [4]

- Hospitalization (inpatient treatment)
“A treatment where a hospital patient occupies a bed for at least one night in the course of treatment, examination or observation.” [5]

“Inpatient care refers to medical treatment that is provided in a hospital or other facility and requires at least one overnight stay. Inpatient care tends to be directed towards more serious ailments and trauma that require one or more days of overnight stay at a hospital.” [6]

1.1.2 VU University Medical Center
VU University Medical Center, or VUmc, is a hospital with all sorts of treatment units. The departments of interest in this research are one day treatment (or outpatient) units; the unit for oncology (DBU OHL) and the unit for internal medicine (DBU ALG). These one day treatment units aim at completing the treatment of each patient within one day, the ‘everything-in-one-day’ principle. DBU OHL and DBU ALG can perform multiple treatments for which hospitalization is not necessary, e.g. chemotherapy, blood transfusions, bone marrow aspirates, and immunotherapy. VUmc introduced the ‘everything-in-one-day’ principle based on considerations towards patient friendliness, i.e. the most comfortable option for the patient.
1.2 Problem statement
VUmc made an organizational decision in July 2016, when it merged the two treatment units DBU OHL and DBU ALG. Before the merger, each unit had their own characteristics, e.g. number of beds available throughout the day, number of nurses available throughout the day, and opening hours. However, VUmc noticed that the two separate units were not functioning as well as required.

Two of the mentioned problems were
- Not enough beds at DBU OHL and unused bed capacity (≠ staff capacity) at DBU ALG;
- Peak times in the arrivals of patients (sometimes four patients at once for one nurse).

This led to
- Waiting times, delay, transfers of patients to the hospital after the opening hours of the DBU;
- Less time for psychosocial care;
- Peaks and variability in the utilization of the beds at the DBU.

As VUmc desires to have a better distribution and coordination of capacity, as well as being able to anticipate on developments within different specializations, they decided to merge DBU OHL and DBU ALG. With this merger, VUmc hoped to create a more balanced workload for nurses, a less regulatory burden for planners and managers, and enable growth of production.

Evidently, the planning after the merger is very different from the planning before the merge. Hence, VUmc is now interested in how to optimize the planning of the new (combined) unit, with an optimal balance between the utilization of existing beds and staff, and robustness against disturbance (e.g. emergency patients arriving, lab results, delay earlier in the chain/throughout the day). VUmc is also interested in some what-if scenarios; what is the influence of changing opening hours and capacity utilization throughout the day on productivity?

The focus of this research paper is twofold, a) the development of a mathematical model to help with calculations for what-if scenarios and b) creating an optimization model for the scheduling system.

1.3 Outline of paper
The structure of this research paper is as follows. It starts with a discussion of the characteristics of each DBU and the approach towards the stated problem in Section 2. Section 3 discusses the methods and materials that are used to create the optimization model. Section 4 will show the results and interpretations. Finally, this research paper ends with a conclusion in Section 5 and some limitations and recommendations concerning the model in Section 6.
2 Description of the one day treatment units
This section gives a description of the one day treatment units for which this research is constructed. Subsection 2.1 gives a short overview of the data provided by VUmc. As this research deals with a merge of two separate outpatient units, subsection 2.2 describes the characteristics of each separate unit, followed by the assumptions made for an optimal planning. Then subsection 2.3 describes some important decision that were made concerning patient groups and the problem statement.

2.1 Data description
In order to investigate and give as much information as possible about the current situation at the one day treatment units, VUmc has provided two datasets on May to October 2016 for this study. One concerning the scheduled information only and one where the scheduled data is coupled with admission data. This second dataset contains 7667 lines, where each line holds information about a unique appointment. Hence, each line holds the planned duration and the actual duration of the appointment, the planned time and actual time of arrival, the date of the appointment, at which unit the treatment took place, what type of appointment was planned and which bed the patient was placed on.

It is important to mention that the scheduled data was coupled to the admission data later on in the process of this study. This results in the fact that it is not very clear whether more appointments are scheduled than that are actually taking place. On top of that, some lines in the data only hold actual admission data (± 0.2% of the lines), because the correct combination between admission information and scheduled information could not be found. Furthermore, currently the scheduled information can state that a no show occurred, however, the actual information shows that a treatment has taken place. Luckily, this only occurs in 175 instances, which is only 2.3% of the 7667 lines, and thus it is decided to ignore this. Finally, some lines were removed from the data as the appointments took longer than one day, which does not fit within the ‘everything-in-one-day’-principle for these treatment units. This resulted in a data file containing 7467 lines with information. When it comes to the no show data and the data with appointments longer than one day, it might be interesting to look into whether it concerns erroneous records. However, this is outside the scope of this research.

As the minimum, average and maximum number of arrivals per day per treatment unit are important during this study Table 1 shows these numbers. For DBU OHL it can be seen that the overall minimum and maximum are 31 and 68, respectively. Whereas for DBU ALG these numbers are equal to 1 and 18 respectively. The average number of patients treated per day at DBU OHL is equal to 50 and the average number of patients treated at DBU ALG is equal to 11.

<table>
<thead>
<tr>
<th>Day</th>
<th>DBU OHL</th>
<th>DBU ALG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min. no. treated</td>
<td>Average no. treated</td>
</tr>
<tr>
<td>Monday</td>
<td>37</td>
<td>48</td>
</tr>
<tr>
<td>Tuesday</td>
<td>37</td>
<td>47</td>
</tr>
<tr>
<td>Wednesday</td>
<td>49</td>
<td>56</td>
</tr>
<tr>
<td>Thursday</td>
<td>44</td>
<td>54</td>
</tr>
<tr>
<td>Friday</td>
<td>31</td>
<td>43</td>
</tr>
</tbody>
</table>

*Table 1 Patients treated per day, DBU OHL & DBU ALG*
2.2 Characteristics & assumptions
A mathematical model is a description of a system using mathematical concepts and language. Such a model is extensively used by analysts to study the effect of different components in real situations. Hence, a model is a simplified representation of such situations. To be able to create a model, some characteristics and assumptions on the current situation at VUmc are discussed in this subsection.

2.2.1 Characteristics
- **DBU OHL**
  DBU OHL is mainly located on the fifth floor of the multiple floored outpatient clinic of VUmc. This unit is open every weekday; on Monday and Thursday the opening hours are from 08:00 to 18:00 and on all other weekdays the opening hours are from 08:00 to 17:00. The unit holds 23 beds and chairs and 1 so called ‘bone marrow-room’ plus 4 AKZI-beds on the fourth floor. An AKZI bed is a special type of bed, where intensive treatment takes place, e.g. stem cell transplantation, and for which a doctor should always be present. The unit thus holds 28 available ‘beds’ per day in total. Furthermore, the unit aims to have enough nurses available throughout the day. Each day of the week, all nurses get assigned an average of 4 beds each, for which he/she is responsible. Finally, the total number of admissions of this unit per day lies between 60 and 80 according VUmc. However, from Table 1 it can be seen that the average number of patients treated per day lies below 60; an average of 50 patients per day. This difference might be of practical importance; however, this aspect is outside the scope of this research and will have no influence on the model that is introduced later on.

- **DBU ALG**
  DBU ALG is located on the fourth floor of the outpatient clinic of VUmc. The fourth floor is open on Monday through Wednesday and on Friday, always from 08:00 until 17:00. Usually, this unit is closed on Thursdays, however, in the received data the unit has been open on seven Thursdays in the period from May until October. Finally, the unit has access to 5 beds and every day enough nurses will be available to treat all patients.

2.2.2 Assumptions for an accurate planning
Since the ‘everything-in-one-day’ principle is very important for VUmc, it is important that planners make an accurate planning in which this principle can be achieved. To be able to make such a planning for this unit, the following assumptions should be made.
- For each unit, patients should be planned during the opening hours, which are:
  - For DBU OHL these planning times are
    - Monday and Thursday from 08:00 – 17:30
    - Tuesday, Wednesday and Friday from 08:00 – 16:30
  - For DBU ALG these planning times are
    - Monday, Tuesday, Wednesday, Friday from 08:00 – 16:30
- The planners are only able to plan patients on the total number of beds/chairs mentioned previously
- It is assumed that planners do not plan multiple appointments at the same time on the same bed
2.3 Patient groups
Since this research concerns two separate outpatient units that were merged into one, the research started with an investigation of each unit separately.

Out of the 7647 lines of received data 6449 of the lines concern the DBU OHL department, i.e. approximately 84% of the data is on DBU OHL. Thus, the decision was made to initially focus on doing a performance analysis and creating a model for this department only. The calculations and implementation for DBU ALG will follow the same procedure.

Secondly, to create a simplification of reality, it is necessary to create groups for all patients as there are a lot of different characteristics to a patient. Initially, the decision was made to create groups based on the type of appointment and their average actual duration. After consulting with VUmc, this idea was altered to creating groups based on the actual average duration of an appointment and the bed the appointment is planned on. For DBU OHL this led to a categorization of patients into 5 groups, where each group holds multiple bed types,

- Group 1 all appointments with an actual duration (S) between 0 and 120 minutes
- Group 2 all appointments with 120 < S \leq 150 minutes
- Group 3 all appointments with 150 < S \leq 180 minutes
- Group 4 all appointments with 180 < S \leq 230 minutes
- Group 5 all appointments with S > 230 minutes

Since less data is available for DBU ALG all patients are categorized into 1 group, i.e. a group of patients with an appointment with an actual duration longer than or equal to 120 minutes. Appendix A shows the bed title assigned to each group.

Finally, as seen before, patients can be categorized based on several characteristics. This paper assumes the average actual duration and the type of bed an appointment is planned on. However, these characteristics might change, as the desire of VUmc on how to categorize their patients might alter. To make sure that the ‘product’ created during this research can still be used in that case, a Macro is designed within Excel, that can handle user-defined groups. This Macro consists of two buttons, i.e. “Create DBU OHL output” and “Create DBU ALG output”. When either one of these buttons is clicked, the Macro performs multiple tasks for the desired department, namely

- It starts counting the number of arrivals per interval per day per group
- It counts the number of appointments with pre-specified duration

The outcomes of the two tasks performed by the Macro will later be used for a queueing model.
3 Methods and materials
This section describes the methods and materials used to create an optimization model for a planning of the merged situation. Moreover, this model can be used in more general situations. As the situation after the merge can be modelled by a queueing model, some useful queueing theory is explained in subsection 3.1. Subsection 3.2 describes a performance analysis of the situation after the merge. And in subsection 3.3 an optimization model will be introduced.

3.1 Queuing theory
Queueing theory deals with problems in which clients arrive and leave a system with servers that involve queueing (or waiting). Typical examples might be supermarkets (a person who waits for a free cashier to pay for his/her groceries), computers (list of the to be printed documents), and public transportation (waiting for a train or bus). In this research paper the situation within the merged facility will be treated as a queueing system.

In queueing theory, $M/G/\infty$ represents a model where arrivals are assumed to be Markovian (a Poisson process with parameter $\lambda(t) = \lambda$, for all $t$, where $t$ stands for a time epoch), the service times have a general distribution and there are infinite many servers. Such a model is of relevance for this study; the virtual infinite capacity can be used as an ideal situation and the $M/G/\infty$ can model the demand for care. The offered load in this model can be written as $\rho = \lambda \beta$, with $\beta = \text{ES}$ (expected service time). In this model, each arrival will immediately be assigned to a server as there are infinite many servers. Hence, the number of occupied servers is equal to the number of present clients in the system. Then, the probability that $k$ servers are occupied is given by

$$p_k = \mathbb{P}(X = k) = e^{-\rho} \frac{\rho^k}{k!},$$

with $X =$ number of occupied servers in steady state.

An $M/G/s/s$ (or Erlang loss) model represents a queueing system where arrivals are assumed to be Markovian, the service times have a general distribution, there are $s$ servers and no waiting positions. Since there are no waiting positions, evidently, if a client finds no server available upon arrival, this client leaves the system and will be lost. The probability that $k$ servers are occupied is given by

$$p_k = \mathbb{P}(X = k) = \frac{\rho^k / k!}{\sum_{n=0}^{s} \rho^n / n!}.$$  

Multiplying the numerator and denominator of this probability with $e^\rho$ gives

$$p_k = \mathbb{P}(X = k) = \frac{\rho^k / k!}{\sum_{n=0}^{s} \rho^n / n!} \cdot \frac{e^{-\rho}}{e^{-\rho}}$$

which corresponds to

$$p_k = \mathbb{P}(X = k) = \frac{\mathbb{P}(X = s)}{\mathbb{P}(X \leq s)},$$

with $X_\infty =$ number of beds for infinite many servers. This shows that the $M/G/\infty$ model can be used as an approximation for determining the blocking percentages when there are $s$ servers instead of infinite many servers. [7]
When the above is interpreted for this specific research, we get the following model description. The arrivals of patients can be assumed to be time-dependent and according a Poisson process, then the model for this scenario can be written as $M_t/G/s/s$. As the arrival process is considered to be time-dependent, this model is non-stationary. The $M_t/G/s/s$ model can be approximated by replacing the finite capacity with infinite capacity, i.e. creating a $M_t/G/\infty$ model. Green et al. [8] show how to adapt stationary queueing models for use in nonstationary environments, such that time-dependent performance is captured, and staffing requirements can be set. It deals with time-dependent arrivals by applying the modified offered load (MOL) approximation. This approximation can be explained as follows, the distribution of the number of busy servers in the $M_t/G/s/s$ queue at time $t$ is approximated by the steady-state distribution of the stationary $M/G/s/s$ queue with an offered load (arrival rate times mean service time) equal to the mean number of busy servers in the $M_t/G/\infty$ queue at time $t$. [9]

### 3.2 Performance analysis

Creating an optimal planning for the merged unit is the one of the aims of this study. However, in order to state whether a planning is optimal, one should be able to evaluate a specific planning (conduct a performance analysis). In this case, the performance analysis consists of determining the required capacity, the variance of this capacity and the expected shortage of beds per time epoch. The $M_t/G/\infty$ model is used to determine these three performance measures.

The expected number of occupied beds at time $t$ is, according to the $M_t/G/\infty$ model,

$$
\mathbb{E}[X(t)] = m(t) = \sum_{u=0}^{\infty} \lambda(u) \mathbb{P}(S > t - u)
$$

with $X(t) =$ number of occupied beds at time $t$, $\lambda(t) =$ expected number of arrivals at time $t$ and $\mathbb{P}(S > t - u) =$ probability that a patient stays more than $t - u$ intervals. The $\lambda(t)$ are determined based on the provided data. All arrivals of a particular group, on a specific day, and specific interval are counted by the Macro described in subsection 2.3. $S$ is also calculated based on the data, i.e. the Macro counts all appointments with a specific duration. Since the distribution of the length of stay of a patient does not show similarities with a well-known distribution, it is decided to use an empirical distribution for the duration of an appointment.

As the number of patients in the $M_t/G/\infty$ queue follows a Poisson distribution, it directly follows that the variance equals the mean, i.e. $\text{Var}[X(t)] = \mathbb{E}[X(t)] = m(t)$. In general, when scheduling appointments, the variability in the number of occupied beds can be reduced by admitting exactly $\lambda(u)$ patients at time $u = 0, 1, ..., T$, instead of a Poisson random variable with rate $\lambda(u)$. In that case, the number of patients present would follow a mixed binomial distribution, having the same mean as above and variance

$$
\text{Var}[X_{\text{scheduled}}(t)] = \sum_{u=0}^{\infty} \lambda(u) \mathbb{P}(S > t - u)(1 - \mathbb{P}(S > t - u)).
$$

For the unit under study, patients are scheduled on a very short notice; the absence of a substantial waiting list makes it difficult to follow a tight schedule for the admitted number of patients per time unit. Since many other factors influence the scheduling process as well, a conservative approximation is used. That is, in the remainder of this paper, it is assumed that the number of scheduled patients follow a Poisson random variable. Consequently, it holds that $\text{Var}[X(t)] = \mathbb{E}[X(t)] = m(t)$. 

The expected shortage of beds can then be calculated by the following formula
\[
\mathbb{E}[(X(t) - B(t))^+] = m(t) \mathbb{P}(X(t) \geq B(t) - 1) - B(t) \mathbb{P}(X(t) \geq B(t)),
\]
with \(X(t) = \text{number of occupied beds at time } t\), \(\lambda(t) = \text{expected number of arrivals at time } t\), \(B(t) = \text{number of available beds at time } t\) and \(\mathbb{P}(X(t) > B(t)) = \text{probability that the number of occupied beds is larger than the number of beds available at time } t\). It is important to mention, that \(X(t)\) is assumed to be Poisson distributed with parameter \(m(t)\).

**Derivation of the expected shortage of beds per time epoch:**

The expected (hence average) shortage can be formulated by either one of the following formulas based on the probability distribution of \(X(t)\), the number of occupied beds at time \(t\).

**When \(X(t)\) has a continuous probability distribution,**
\[
\int_{y=B(t)}^{\infty} (y - B(t)) f_{X(t)}(y) \, dy = \mathbb{E}[(X(t) - B(t))^+]
\]

**When \(X(t)\) has a discrete probability distribution,**
\[
\sum_{y=B(t)}^{\infty} (y - B(t)) \mathbb{P}(X(t) = y) = \mathbb{E}[(X(t) - B(t))^+]
\]

Thus, in order to be able to calculate the expected shortage of beds, one should know the probability distribution of \(X(t)\). During this study, \(X(t)\) is assumed to be Poisson distributed, as this distribution describes the probabilities of a particular number of sporadic events within a given time interval. Thus, \(X(t)\) is Poisson distributed in each interval with parameter \(m(t)\), the expected number of occupied beds.

The Poisson distribution is a discrete distribution, hence \((ii)\) should be used to calculate the expected shortage of beds in time epoch \(t\). Thus, the expected shortage is calculated with \(X(t) \sim \text{Poisson}(m(t))\).

\[
\mathbb{E} \left[ (X(t) - B \leq (t))^+ \right] = \sum_{y=B(t)}^{\infty} (y - B(t)) \mathbb{P}(X(t) = y) = \sum_{y=B(t)}^{\infty} \frac{m(t)y}{y!} - B(t) \mathbb{P}(X(t) \geq B(t))
\]

**3.3 Optimization model**

In this subsection, an optimization model for the planning of the merged DBU is introduced. This model is based on the optimization models introduced by Adan et al. [10] and Bekker et al. [11] for the planning of hospital admissions. It is important to mention that both references discuss their implementations for clinics and not one day treatment units.
Adan et al. [10] investigate the influence of using a stochastic instead of a deterministic length of stay. It discusses a mixed integer linear programming model developed for planning with stochastic length of stay. The results obtained by the model indicate that the produced schedules have a better performance on target utilization levels of resources. Bekker et al. [11] analyse the impact of the variability in admissions and lengths of stay on the required amount of capacity, and determine admission quota for scheduled admissions to regulate the occupancy pattern. Given a structural weekly admission pattern, they apply a time-dependent analysis to determine the mean offered load per day. This is combined with a Quadratic Programming model, which helps determine the optimal number of admissions per day, in a way that an average desired daily occupancy is achieved.

From the previous, it can be concluded that the occupation of beds throughout the day is linear in the expected arrivals per interval, i.e. $\mathbb{E}[X(t)] = m(t) = \sum_{u=0}^{t} \lambda(u) \mathbb{P}(S > t - u)$ is linear in $\lambda(t)$. This characteristic can be used to formulate a linear programming problem, which creates a planning for the admission of outpatients.

### Notation
Table 2 shows the notation for the linear programming problems, which will be specified later.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{E}[X(t)]$</td>
<td>Average number of occupied beds in interval t</td>
</tr>
<tr>
<td>$\lambda(t)$</td>
<td>Number of arrivals/admissions in interval t</td>
</tr>
<tr>
<td>$B_i(t)$</td>
<td>Number of beds in interval t</td>
</tr>
<tr>
<td>$\Lambda_t$</td>
<td>Total number of arrivals/admissions</td>
</tr>
<tr>
<td>$a$</td>
<td>Max. number of arrivals per time epoch (based on available nurses)</td>
</tr>
<tr>
<td>$e$</td>
<td>Max. average number of occupied beds after opening hours (final occupation)</td>
</tr>
<tr>
<td>$w$</td>
<td>Weight between 0 and 1</td>
</tr>
<tr>
<td>$T$</td>
<td>Interval after opening hours, i.e. last interval</td>
</tr>
</tbody>
</table>

### Assumptions
Since the linear programming problems should come up with a planning for outpatients, some assumptions should be made.

- **Opening hours:**
  - DBU OHL: Monday and Thursday from 08:00 – 18:00
    Tuesday, Wednesday and Friday from 08:00 – 17:00
  - DBU ALG: Monday, Tuesday, Wednesday and Friday from 08:00 – 18:00
    Closed on Thursday

- **Hours that arrivals can occur:**
  - DBU OHL: Monday and Thursday from 08:00 – 17:30
    Tuesday, Wednesday and Friday from 08:00 – 16:30
  - DBU ALG: Monday, Tuesday, Wednesday and Friday from 08:00 – 16:30
    Closed on Thursday

- Day is divided into intervals of 30 minutes;
- After opening hours all DBU beds should be empty or smaller or equal to $e$;
- Five patient groups for DBU OHL, only one group of patients for DBU ALG.
\textbf{Linear programming problem for DBU OHL}

\begin{align*}
\text{min} & \quad \sum_{i=1}^{5} w_i + (1 - w)z \\
\text{s.t.} & \quad E[X(t)] \leq zB(t), \quad t = 0, \ldots, T - 1 \\
& \quad E[X(T)] \leq e, \\
& \quad E[X_i(t)] \leq z_i B_i(t), \quad i = 1, \ldots, 5 \text{ & } t = 0, \ldots, T \\
& \quad \sum_{t=0}^{T} \lambda(t) = \Lambda, \quad t = 0, \ldots, T \\
& \quad \lambda(t) \leq a, \quad t = 0, \ldots, T - 1 \\
& \quad \sum_{t=0}^{T} \lambda_i(t) = \Lambda_i, \quad i = 1, \ldots, 5 \text{ & } t = 0, \ldots, T \\
& \quad \lambda_i(t) \geq 0, \quad i = 1, \ldots, 5 \text{ & } t = 0, \ldots, T - 1 \\
& \quad \lambda(T) = 0, \quad i = 1, \ldots, 5 \\
\end{align*}

In this LP-problem the objective function minimizes the maximum number of occupied beds with the help of the introduced variable \( z \). Hence, the objective is to minimize the occupancy. The \( z \) variable indicates the average maximum number of occupied beds. When \( z \) is larger than 1 not enough bed capacity is available no matter which admission pattern is applied. Furthermore, the objective function holds a constant, namely \( w \) with \( w \) in \([0, 1]\). This \( w \) indicates the importance towards which problem to solve:
- when \( w = 0 \) the unit is stabilized as a whole;
- when \( w \) is equal to 1 the problem is solved by group and 5 separate MILP are solved;
- or a little bit of both, where \( w \) indicates what is more important.

The actual number for this weight can be determined by the user dependent on the preference towards solving the problem.

The first constraint restricts the number of expected occupied beds per time epoch to the number of available beds per time epoch \( B(t) \) times \( z \), per group as in total. Hence, the number of occupied beds should be smaller or equal to the number of available beds. The variable \( e \) indicates the maximum final average bed occupancy. Moreover, the sum of all admissions is equal to target number of admissions \( \Lambda \). This holds both in total as per group. The number of arrivals per time epoch are larger or equal to 0 and the total number of arrivals per time epoch have an upper bound indicated by variable \( a \). Finally, there can be no admissions in the last interval.

\textbf{Linear programming problem for DBU ALG}

\begin{align*}
\text{min} & \quad z \\
\text{s.t.} & \quad E[X(t)] \leq zB(t), \quad t = 0, \ldots, T - 1 \\
& \quad E[X(T)] \leq e, \\
& \quad \sum_{t=0}^{T} \lambda(t) = \Lambda, \quad t = 0, \ldots, T \\
& \quad \lambda(t) \leq a, \quad t = 0, \ldots, T - 1 \\
& \quad \lambda_i(t) \geq 0, \quad i = 1, \ldots, 5 \text{ & } t = 0, \ldots, T - 1 \\
& \quad \lambda(T) = 0 \\
\end{align*}

For this linear programming problem, the same interpretation of the model holds as DBU OHL.
General notifications

It is important to mention that the LP problem for the merged situation is in general the same as the linear programming problem for DBU OHL. The only difference in this case is that there are six patient groups instead of five (thus, $i = 1, \ldots, 6$).

With the described linear programming problem, a MILP is solved where $\lambda_i(t)$ should be integer. However, a relaxation of the model is created in which the requirement of integer solutions is released. To come up with integer solutions of the problem a planning heuristic is performed. This heuristic can be formulated as follows

$$\lambda_i(t) = \left[ \sum_{n=1}^{t} \lambda_i(n) \right] - \sum_{n=1}^{t-1} \bar{\lambda}_i(n)$$

Finally, the above LP-problems are solved using the Solver Tool of Excel.
4 Results
In this section, the results of this study are displayed and interpreted. Subsection 4.1 shows a performance analysis on the situation before the merger in the separate one day treatment units, DBU OHL and DBU ALG. Subsection 4.2 shows the results of the optimization model for both each separate DBU and the results after the merger. Subsection 4.3 describes the results of a few what-if scenarios.

4.1 Performance analysis
This subsection gives the expected number of occupied beds and the expected shortage of beds per time epoch for each unit. In subsection 3.2 the following formulas were introduced to calculate these performance measures

\[
\text{Expected number of occupied beds} = \mathbb{E}[X(t)] = m(t) = \sum_{u=0}^{t} \lambda(u) \mathbb{P}(S > t - u)
\]

\[
\text{Expected shortage of beds} = \mathbb{E}[(X(t) - B(t))^+] = m(t) \mathbb{P}(X(t) \geq B(t) - 1) - B(t) \mathbb{P}(X(t) \geq B(t))
\]

4.1.1 DBU OHL
The one day treatment unit DBU OHL is the facility for oncology and holds 28 beds in total, as mentioned in subsection 2.2. Subsection 2.3 mentioned that the arrivals of DBU OHL are split into five patient groups. The results shown in this section are based on the total, thus the sum of these five patient groups.

Figure 1 gives insight in the expected number of occupied beds during a day based on the provided data. It can be concluded that before the merge, in general, Wednesday was the busiest day. Furthermore, as the nurses of DBU OHL go on lunch break between 12:00 and 14:00, which means that half of the staff is available during these hours, it is required that the number of expected occupied beds is lower. From the figure, it can be seen that the expected capacity during lunch break actually is lower, which is a positive aspect to the used planning.
The current planning of DBU OHL arrivals is not as bad as expected, this can be seen from Figure 1. However, two aspects which might need some attention, are

- The average bed occupancy during the day is not stable. The unstable bed occupancy could be a problem for the one day treatment unit. From the figure, it can be seen that the day starts off slowly and has to deal with peaks just before and right after lunch break for most days;
- The occupancy at the end of each day is not equal to zero. For the one day treatment unit to complete the treatment of each patient within one day (the ‘everything-in-one-day’ principle), no beds or fewer beds should be occupied at the end of the day.

These two aspects are hard to prevent, and therefore, for this research it could be concluded, based on the data, that the current way of planning is fairly good and no bottleneck based on bed capacity exists.

![Figure 2 Expected number of occupied beds & expected shortage of beds for DBU OHL on Wednesday](image)

<table>
<thead>
<tr>
<th>Time</th>
<th>Expected shortage</th>
<th>Time</th>
<th>Expected shortage</th>
<th>Time</th>
<th>Expected shortage</th>
<th>Time</th>
<th>Expected shortage</th>
</tr>
</thead>
<tbody>
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<td>16:45:00</td>
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</tr>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

*Table 3 Expected shortage of beds on Wednesday for DBU OHL*
During the performance analysis, the expected shortage of beds is also calculated. Figure 2 and Table 3 show the outcome of this for Wednesday, the busiest day at DBU OHL (graphs for all other weekdays can be found in Appendix B). As can be seen from both the table and the figure the expected shortage of beds is very low during Wednesday. This also holds for all other days of the week. The low expected shortage of beds can be explained as the average demand for care lies around 20 whereas 28 beds are available during the day. Hence, based on the available data, there is not a large capacity problem concerning the number of available beds.

4.1.2 DBU ALG
In this subsection, the results for the one day treatment unit for internal medicine, DBU ALG, are interpreted. As mentioned in subsection 2.2, this unit holds only 5 beds and contains only one patient group.

Figure 3 shows the expected number of occupied beds during each day of the week for DBU ALG. As mentioned before, this unit is normally closed on Thursdays. However, the data holds information about seven Thursdays in the period May to October 2016, that is why a line is shown for Thursday. The figure shows that for DBU ALG, in most cases, the same conclusions about bed occupancy can be drawn as DBU OHL; the figure shows a decrease in occupied beds during the lunch break, the average bed occupancy during the day is unstable and not all beds are empty at the end of the day.

The expected shortage of beds at DBU ALG on Mondays can be found in Figure 4 and Table 4 (results for all other days can be found in Appendix B). From this figure and this table, it can be assumed that as there are less beds available than DBU OHL, a shortage of beds is more likely to occur. An average shortage of 0.5 beds on a total of 5 beds is not negligible. However, the numbers are still considerably small and therefore, it might be concluded that there is no bed capacity problem based on data.
Figure 4 Expected number of occupied beds & expected shortage of beds for DBU ALG on Monday

<table>
<thead>
<tr>
<th>Time</th>
<th>Expected shortage</th>
<th>Time</th>
<th>Expected shortage</th>
<th>Time</th>
<th>Expected shortage</th>
<th>Time</th>
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<td></td>
</tr>
</tbody>
</table>

Table 4 Expected shortage of beds on Monday for DBU ALG

4.2 Optimization model

The results of the optimization model mentioned in subsection 3.3 are discussed here. First, the results for each DBU are shown. Followed by the results for the situation after the merger.

4.2.1 DBU OHL

By now, it is known that the patients of DBU OHL are categorized into five groups, see subsection 2.3. Since the patients are categorized into these groups, it is interesting to see whether the length of stay of each patient group has an influence on the optimal planning. Questions such as ‘Should we plan patients with a long length of stay at the beginning of the day or not?’ can be answered by looking at the results of the LP-problem for this unit.

As mentioned in subsection 3.3, the constant weight $w$ of the LP-problem for this one day treatment unit can be varied by the user. Here we only show the results of the LP-problem when $w = 0.1$, which indicates that the LP-problem solves with more importance to the system as a whole. This weight is chosen as this resulted in the best combination between the $z$-values and the minimum value for the decision variable (Appendix C, subsection 8.3.1 contains a table with outcomes for different values of $w$).
Figure 5 shows an optimal planning for 50 patients (average number of arrivals found in the provided data) as well as the expected number of occupied beds throughout the day (right axis). For this planning, the maximum number of arrivals per time epoch is set to 6 (based on the available nurses, $a = 6$) and the final occupation is equal to 3 ($e = 3$). The number of available beds per time epoch is equal to 28 during the day, that is $B(t) = 28$ for $t = 0, \ldots, T$. From the figure, it can be seen that the arrivals of patients with a long length of stay (group 4 and group 5) should occur at the beginning of the day. Apparently, planning patients with a long length of stay at the beginning of the day does have an influence on the total number of occupied beds throughout the day. Finally, as the unit holds 28 beds, it can be concluded that overall enough beds are available at DBU OHL for planning 50 patients during a day.

For each planning produced by the LP-problem, the expected shortage of beds is also calculated. The outcome of the expected shortage of beds for the planning showed in Figure 5 can be found in Appendix C, subsection 8.3.1.

![Planning DBU OHL, with 50 patients](image)

**Figure 5 Planning for DBU OHL, with 50 patients, $a = 6$ & $e = 3$**

### 4.2.2 DBU ALG

The one day treatment unit DBU ALG consists of only one patient group. The optimal planning of arrivals and the expected number of occupied beds with this planning for DBU ALG can be seen in Figure 6. With this schedule, a total number of 11 patients (average number of patients treated per day, found in provided data) are planned per day and it is assumed that the maximum average number of occupied beds after opening hours is equal to 1 ($e = 1$). Furthermore, the maximum number of arrivals per time epoch is set to 2 ($a = 2$).
The z-value with this schedule is equal to 0.680, which indicates that the maximum occupancy during the day on average is equal to 68%. Hence, there are enough beds at the unit to treat all planned patients upon arrival. To have a better indication whether or not the bed capacity is sufficient, one should look at the expected shortage of beds. For this planning the expected shortage of beds can be found in Appendix C, subsection 8.3.2.

### 4.2.3 Situation after merger

Before the results of a possible planning for the merged situation are discussed, a few important variables should be mentioned:

- The arrivals of DBU ALG are seen as an extra patient group to the LP-problem introduced for DBU OHL, hence DBU ALG is group 6;
- In the merged situation 33 beds are available until 17:00, namely the 28 beds of DBU OHL and the 5 beds of DBU ALG. DBU ALG closes at 17:00, thus after 17:00 28 beds are available;
- The maximum average number of occupied beds after opening hours is equal to 3;
- The maximum number of arrivals per time epoch is equal to 6;
- The number of patients planned on a day is equal to 61, the sum of the averages of DBU OHL and DBU ALG;
- Once again, w is equal to 0.1.

Figure 7 shows an optimal planning for each patient group when a total of 61 patients are planned for one day. From this figure, it can be concluded that it is optimal to plan patients with a longer length of stay at the beginning of the day. This way at the end of the day the maximum average number of occupied beds is not larger than a specific number, in this case 3.
Moreover, the figure shows that most arrivals of group 6 (DBU ALG patient group), with a duration longer than 120 minutes, occur at the end of the day, from 13.30 and onwards. Especially, the patients arriving at 16.30 of this patient group might lead to occupied beds after opening hours. This is possible, as the number of occupied beds after opening hours may be equal to 3.

Figure 7 also displays the number of available beds versus the expected number of occupied beds throughout the day (green line, right axis). From this, it can be concluded that the bed capacity is sufficient throughout the day for this planning. This is also further investigated by calculating the expected shortage of beds, which can be found in Figure 8. The same conclusion can be drawn from this figure; the number of beds available is adequate for this planning.
4.3 What-if scenarios
The optimization model, that creates an optimal planning for the situation after the merger, can be used to do some calculations for what-if scenarios. This subsection shows the effects on the optimal planning when the following scenarios are applied:

- The maximum average number of occupied beds after opening hours (final occupation) is restricted to zero;
- The maximum number of arrivals per time epoch is variable; what if less patients can arrive during a time epoch?
- The total number of beds available at a time epoch is variable; less beds available at the beginning of the day, during lunch break and at the end of the day.

Obviously, more what-if scenarios can be analysed; combinations of the above scenarios are also possible.

4.3.1 No occupied beds after opening hours
From a mathematical point of view, it is obvious that not occupied beds after opening hours, $e = 0$, is not desirable, as it represents an expected occupancy. It would be more logical to let the variable $e$ be equal to 0.1 or 0.5. From a practical point of view, the scenario $e = 0$ might be relevant, and therefore is mentioned in this section.

As the one day treatment unit aims at completing the treatment of all patients within one day, it is interesting to look at the possibility of having all beds empty after opening hours. Hence, $\bar{E}[X(T)] \leq e$, where $e = 0$. However, after implementing this and trying to solve the problem for planning 61 patients on a day, with $a = 6$ (max. arrivals per time epoch) and number of available beds as described in subsection 4.2.3, the Solver could not find a feasible solution. The Solver was able to create a planning for a maximum of 56 patients, where all beds are empty (i.e. $e = 0.5$) at the end of the day.

Now, it is interesting to investigate which alterations should be made in order to be able to plan 61 patients during one day and have approximately all beds empty at the end of the day ($e = 0.5$). Increasing the maximum number of arrivals per time epoch (which might mean increasing personnel capacity) gave satisfactory results. The Solver could find a feasible solution, by increasing the maximum number of arrivals per time epoch with only two, hence, a total of 8 arrivals per time epoch. The planning of this scenario can be found in Appendix C, subsection 8.3.3.

When VUmc allows one bed to be occupied at the end of the day, which means only one transfer to another department, a planning is possible for sixty-one patients with a maximum number of arrivals per time epoch equal to six. The maximum number of arrivals could even be decreased to five per time epoch. Which could indicate that less personnel is required, or personnel is less busy throughout the day and more personal attention could be provided to a patient. The planning of this scenario can be found in Appendix C, subsection 8.3.3.

4.3.2 Variable maximum number of arrivals per time epoch
From Figure 7 and Figure 8 in subsection 4.2.3 it is seen that the current bed capacity is sufficient for planning sixty patients a day. But what happens to the planning when less arrivals per time epoch can occur.
Based on multiple model calculations, it can be concluded that an optimal planning is possible with a maximum of only four arrivals per time epoch. This could mean that only four nurses are needed to treat sixty-one patients during one day. Hence, with six nurses available that means less pressure on the staff and possibly more time for personal attention for patients. The planning of this scenario can be found in Figure 9. It should be noticed that in this situation it does take longer for the number of patients present to stabilize. This might lead to more occupied beds at the end of the day, which means the final occupancy might be larger than preferred.

4.3.3 Variable available beds throughout the day
When one tries to replicate the reality as much as possible, it is logical that the number of available beds throughout the day can fluctuate. For example, at the beginning of the day not all nurses are available, which means that arriving patients cannot be assigned to all beds. Same holds during lunch break; half of the staff goes on lunch break.

The available beds per time epoch for the scenario discussed in this subsection can be found in Table 5. This number of available beds can completely be defined by the user. The chosen numbers in Table 5 are based on the following:
- Before 08:00 not all beds should be available to plan patients on, in order to prevent nurses from arriving at a waiting room full of patients;
- At the beginning of the day one of the nurses available at DBU OHL is busy preparing medication between 08:00 and 10:00;
- During lunch break (12:00 – 13:30) half of the staff of DBU OHL goes on lunch break, which means that less beds should be available. This way the nurses that are not on lunch break will not have to deal with too large of a workload.
Figure 10 shows the planning for sixty-one patients, when the number of beds at the beginning of each time epoch are as specified in Table 5. The maximum number of arrivals per time epoch is still equal to six. From the figure, it can be seen that when the day is split into two sections, a section representing the day before lunch and section representing the day after lunch, at the beginning of each section the most arrivals occur. Once again, most appointments with a relatively long duration (group 4 and group 5) are planned at the start of the day. The figure also shows that all appointments of group 6 (DBU ALG group, with appointment duration longer than 120 minutes) take place in the second half of the day. Furthermore, during lunch the number of available beds and the expected occupied beds are approximately the same. This indicates that all beds that are available during lunch will be occupied.

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<th>Av. Beds</th>
</tr>
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<td>33</td>
<td>16:30 &lt; t &lt;= 17:00</td>
<td>33</td>
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<tr>
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<td>33</td>
<td>17:00 &lt; t &lt;= 17:30</td>
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<tr>
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<td>20</td>
<td>17:30 &lt; t &lt;= 18:00</td>
<td>28</td>
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</tbody>
</table>

Table 5 Available beds per time epoch
5 Conclusion
The purpose of this study was twofold, a) develop a mathematical model to help with calculations for what-if scenarios, for example varying beds or less arrivals per time epoch and b) create an optimization model for the scheduling system. Thanks to this study, VUmc now has a generic model available which can calculate all kinds of situations. This model will show the required capacity and a planning for each specific situation. During this study, a performance analysis on the current situation at the outpatient units was also performed.

The performance analysis of each separate outpatient unit was performed based on the provided data. It showed that enough bed capacity is available at both units to treat the average number of patients of each unit. The expected shortage of beds showed to be very small for DBU OHL, whereas for DBU ALG this shortage is larger, as less beds are available for this unit. Moreover, the performance analysis showed there is no reason to conclude that the current way of treating patients throughout a day should cause problems for the DBUs. The planning of each unit looked fairly good. Also, less beds are occupied during lunch break, which is good as less nurses are available during this time. During this study, the focus was only on the average number of occupied beds throughout the day. More insight into the current situation can be obtained when the variation in occupied beds throughout the day is taken into account. As investigating the current situation was outside the scope of this research, this analysis was not performed.

Despite the fact that the current ‘experience based’ planning as applied by VUmc is not as bad as expected, an optimization model was created. This model is used to provide solutions for a planning based on LP-problems (each for every unit, and one for the merged situation). These LP-problems make use of several patient groups with different appointment durations. Hence, an insight in a planning strategy based on durations is provided. In all situations, it can be concluded that appointments with a relatively long appointment duration should be planned at the beginning of the day, in order to be able to close the day with as less beds occupied as possible.

Furthermore, the optimization model is used to investigate a few what-if scenarios on the merged situation. This showed that it is hard to make sure that no beds are occupied after opening hours when sixty-one patients should be treated with the current bed and staff capacity. On the other hand, another what-if scenario showed that sixty-one patients could be treated per day when the maximum number of arrivals is only four per half an hour. Regarding the current staff capacity this might lead to less workload for all nurses and to more personal attention for the patients of the DBU. In this scenario, a maximum of three beds might be occupied at the end of the day, which might mean transferring three patients to another department after closing hours. And finally, when bed capacity is varied throughout the day, based on for example less capacity during lunch break, the optimization model still came up with an optimal solution for a planning of sixty-one patients.
6 Discussion
6.1 Limitations
In this study, patient groups are based on the data of the average actual duration of an appointment in combination with the type of bed an appointment was treated on. Creating patient groups based on more characteristics might be more viable for VUmc.

Furthermore, the provided data does not contain information about whether an appointment at the DBU required a consultation meeting prior to the actual treatment. However, for VUmc this is actually a very constraining factor in the current planning and course of the day. Thus, basically, the model built during this study can be used when VUmc decides to let go of the ‘everything-in-one-day’ principle.

Unfortunately, during this study, little time was spent validating the model, i.e. to what extent does the model correspond with reality. This is a limitation which requires further research. Currently, the model is dependent on the provided data. Not much information is available on occurrences of casualties, such as incoming emergency patients, lab results that take longer than expected, or delay earlier in the chain. This makes it hard to take account for in such a model.

Finally, the data provided for this study consists of data from May until October 2016. This means only two months of data on the situation at the DBUs before the merger is provided. Perhaps more data prior to the merger would provide more insights in the actual problem stated by VUmc.

6.2 Recommendations
For the current situation based on the expected number of occupied beds, i.e. average number of beds occupied, it was concluded that the current way of planning is not as bad as it seems. However, as this is only based on the mean, it might also be interesting to look into the variance in occupancy. Furthermore, another aspect of interest for VUmc might be to investigate the difference between experience of the staff, i.e. not enough bed capacity, versus the conclusion of this research that there is no capacity problem based on the expected occupied beds.

As mentioned, it might be interesting to create patient groups based on more characteristics, such as type of treatment, length of treatment and whether an appointment requires a consultation meeting prior to the treatment. This might create a model that can help with more realistic calculations. It should be mentioned that the patient groups should not be overly refined based on too many characteristics.

The model built during this study currently determines the number of patients to plan per patient group based on the ratios extracted from the data. When the patient groups are defined based on multiple characteristics, it might be useful for the user of the tool to be able to state the number of patients per group that should be planned, instead of a total number of patients.

It might also be interesting for VUmc to look into the combination consultation meeting and appointment at the DBU. As stated before, this study only focusses on DBU OHL and DBU ALG. Further research into the combination of consultation and treatment might lead to another point of view towards an actual planning strategy.
7 References


8 Appendices

8.1 Appendix A

**Patient groups**

Based on average actual duration of an appointment on the bed used for that appointment

<table>
<thead>
<tr>
<th>DBU OHL (VDAGON)</th>
<th>Group 1 (0 - 120 minutes)</th>
<th>Group 2 (120 - 150 minutes)</th>
<th>Group 3 (150 - 180 minutes)</th>
<th>Group 4 (180 - 230 minutes)</th>
<th>Group 5 (&gt;230 minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VARMAESDENONGK, LSV</td>
<td>DBU OHL BED 3 VUMC</td>
<td>DBU OHL BED 4 VUMC</td>
<td>DBU OHL BED 10 / VERNEVELLEN VUMC</td>
<td>DBU OHL AKZI BED 2 VUMC</td>
<td>DBU OHL CISPLATIN BED 1 VUMC</td>
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<tr>
<td>DBU OHL BEENVERGPNCTIE VUMC</td>
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<td>DBU OHL BED 7 VUMC</td>
<td>DBU OHL BED 8 VUMC</td>
<td>DBU OHL AKZI BED 4 VUMC</td>
<td>DBU OHL BLOODTRANSFUSIE BED 2 VUMC</td>
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<tr>
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<td>DBU OHL BED 5 VUMC</td>
<td>DBU OHL BED 8 VUMC</td>
<td>DBU OHL BED 11 / VERNEVELLEN VUMC</td>
<td>DBU OHL AKZI BED 3 VUMC</td>
<td>DBU OHL BLOODTRANSFUSIE BED 1 VUMC</td>
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<td>DBU OHL BED 6 VUMC</td>
<td>DBU OHL BED 12 VUMC</td>
<td>DBU OHL HOOFDHUIDERDELING BED 1 VUMC</td>
<td>DBU OHL 4W BED 13 VUMC</td>
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**Patient groups**

Based on average actual duration of an appointment on the bed used for that appointment

<table>
<thead>
<tr>
<th>DBU ALG (VDAGPK)</th>
<th>Group 1 (&gt;120 minutes)</th>
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8.2 Appendix B
8.2.1 Performance analysis, DBU OHL
Expected shortage of beds for all week days
8.2.2 Performance analysis, DBU ALG
Expected shortage of beds for all week days

Monday

Tuesday

Wednesday

Thursday

Friday
8.3 Appendix C

8.3.1 Optimization model, DBU OHL

<table>
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</table>

Table 6 Results for the decision variable and the z-values of the LP-problem varying w, with 50 patients to plan, 28 beds available, \( e = 3 \) & \( a = 6 \)

![Figure 11](image1.png)

Figure 11 Expected occupied beds & shortage of beds for planning DBU OHL, with 50 patients, 28 beds, \( a = 6 \) & \( e = 3 \)

8.3.2 Optimization model, DBU ALG

![Figure 12](image2.png)

Figure 12 Expected occupied beds & shortage of beds for planning DBU ALG, with 11 patients, 5 beds, \( a = 2 \) & \( e = 1 \)
8.3.3 What-if scenarios
No occupied beds after opening hours

![Planning merged situation, with 61 patients](image)

*Figure 13 Planning for merged situation, with 61 patients, $a = 8$ & $e = 0.5$*

![Planning merged situation, with 61 patients](image)

*Figure 14 Planning for merged situation, with 61 patients, $a = 6$ & $e = 1$*